Statement of Purpose

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1 Motivation

My general research interest lies in the theory aspect of Machine Learning, in which we use different kinds of mathematical tools and models to understand and provide guidance to Machine Learning research and practice. As someone who has absolute passion about math, I truly believe the power of math to describe and interpret the problems in this field. In particular, hardness of learning and approximation, generalization error and sample complexities relating to which are all essential topics in machine learning research, and has typically been studied in the finite-dimensional and discrete cases. To some extent, this is true when these simpler cases do capture the essential idea of the solution to real world problems, yet in most cases, general function spaces such as Banach spaces are more powerful in describing the world. Especially in scientific machine learning when people deal with PDEs, we typically need to study function class with extra smoothness structures such as Sobolev spaces. Understanding the infinite-dimensional nature of many machine learning problems also provides insights into developing theoretical interpretation for a broader range of topics such as deep learning theory. **Therefore, I would like to focus on the statistical and learning-theoretic properties of these function spaces in my research in the future. Meanwhile, I'm interested in applying these techniques into the theory and practice of areas such as deep learning and reinforcement learning.**

I am confident in leveraging my outstanding background in mathematics, computer science and physics to solve challenging problems in learning theory, and to contribute novel and profound insights to this research area. I enrolled in several senior courses in mathematical major such as Functional Analysis and Differential Geometry, and I accomplished a perfect score of 100% for the latter. I also audited many mathematical courses ranging from Probability with Martingale theory to Harmonic Analysis, which prepared me with decent mathematical maturity and technical tools to handle challenging theories and proofs. My strong interest in math also drives me to further explore more topics and their potentials to handle even more complex and broader range of problems in the future.

2 Research Experiences

My previous experience in theoretical research explores machine learning from three different perspectives, respectively with focus on the intersection between information theory and statistical learning theory, offline reinforcement learning theory and min-max learning hardness in function spaces. During the process, I worked on distinct models of demonstrating learning problems, which either studies generalization error or sample complexities as its ultimate goal. These experiences allow me to have a general taste of the topics people care about in statistical machine learning community, foster me to become an independent researcher and prepare me with a handful of useful tools and techniques for my future research.

Information-Theoretic Method for Generalization Error. I conducted this research independently under the supervision of Prof. Yuheng Bu. The methodology of this line of work lies in understanding the expected generalization error of stochastic learning algorithms using information measures. With recent results [1, 2, 5] that upper-bound the generalization error using mutual information between the hypothesis W and the dataset S: $I(W;S)$, people discover that a similar bound using information from individual samples Z_i is more desirable, especially that it's much easier to estimate while training comparing to the information of extremely high-dimensional dataset. Our work [3] to be presented in ITW 2024 addressed this issue for Gibbs algorithm by proposing an asymptotic equivalency between $\sum_{i=1}^{n} I_{SKL}(W;Z_i)$ and $I_{SKL}(W;S)$. For asymptotic of individual samples, I mainly used the strong law of large numbers to obtain the exact convergence rate, and for asymptotic of entire dataset, I utilized Khintchine's inequality to sandwich $n \cdot I_{SKL}(W;S)$ for the exact rate. Our further result indicating $I(W;S) \sim L(W;S)$ also addresses the problem of how mutual information and lautum information scales relatively, i.e. $L(W;S)/I(W;S)$, which plays a major role in the convergence rate of generalization error for Gibbs algorithm as shown in Theorem 2 [1].

Off-Policy Evaluation for POMDP Under Smoothness Condition. This is another independent research project of mine with supervision from Prof. Nan Jiang, and is formulated as a research note. By assuming the Lipchitz continuity of value function w.r.t. belief state, I arrived at a PAC sample complexity guarantee using belief space coverage assumption, the general proof follows the process as demonstrated below:

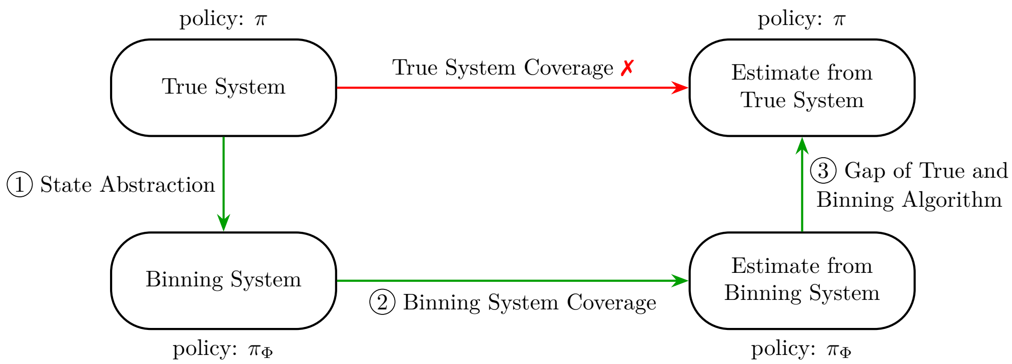

Figure 1: Pipeline of the analysis

Specifically in step 1, I descend the true belief space MDP system to a abstract binned system where the policy π is also descended to an abstract policy π_Φ. Using similar ideas of state abstraction, I bridge the abstraction gap using the size of bins ε. I also show that belief space bisimulation assumption can be replaced by Lipchitz value function using chaining techniques. In step 2, I employed the standard analysis for MDP, with the coverage assumption for the binned belief space, which can be much more tractable than the coverage of the true system due to "The Curse of Horizon". Eventually for step 3, I utilize the Lipchitz property of value function again to control the difference of the binned version and the true version of the same algorithm on the same off-line dataset. Combining all the analysis above, I put forward the final sample complexity guarantee.

During the project, I also investigated the recent Future Dependent Value Function (FDVF) algorithm [4, 6], which is restricted to only memoryless policies because of "The Curse of History". With belief space smoothness and some fast forgetting assumption, we may also arrive at a better sample complexity guarantee by carefully adjusting the observation window, which is a project I'm still working on.

Function Estimation in Banach Spaces. This is an ongoing project with Prof. Yiping Lu, where we focus on the min-max hardness of learning in Banach space. Due to the infinite-dimensional nature of function spaces, we uses tools such as Information-based complexity, Gelfand Width and methods such as duality to investigate these hardness. I also get to see the application of information-theoretic method in the statistical analysis of these types of problem. All of these greatly broaden my skill sets for future research.

Other Empirical Research Experiences. Apart from all my experiences in theoretical research, I've devoted to some projects which testified my ability on empirical tasks as well. Project wise I have done some work in Real-time Ray Tracing, which is a complex coding experience with exposure to very basic interaction between CPUs and GPUs, and I have also built up LBM code from scratch for fluid simulation, which is also a complicated task in scientific computing. I've also conducted experiments on information-theoretic properties of the Gibbs algorithm, of which the results motivated my theoretical research as mentioned above.

References

[1] Yuheng Bu, Shaofeng Zou, and Venugopal V Veeravalli. Tightening mutual information-based bounds on generalization error. *IEEE Journal on Selected Areas in Information Theory*, 1(1):121–130, 2020.

[2] Daniel Russo and James Zou. How much does your data exploration overfit? controlling bias via information usage. *IEEE Transactions on Information Theory*, 66(1):302–323, 2019.

[3] **Youheng Zhu** and Yuheng Bu. Information-theoretic analysis of the gibbs algorithm: An individual sample approach. *arXiv preprint arXiv:2410.12623*, 2024.

[4] Masatoshi Uehara, Haruka Kiyohara, Andrew Bennett, Victor Chernozhukov, Nan Jiang, Nathan Kallus, Chengchun Shi, and Wen Sun. Future-dependent value-based off-policy evaluation in pomdps. In A. Oh, T. Naumann, A. Globerson, K. Saenko, M. Hardt, and S. Levine, editors, *Advances in Neural Information Processing Systems*, volume 36, pages 15991–16008. Curran Associates, Inc., 2023.

[5] Aolin Xu and Maxim Raginsky. Information-theoretic analysis of generalization capability of learning algorithms. *Advances in neural information processing systems*, 30, 2017.

[6] Yuheng Zhang and Nan Jiang. On the curses of future and history in future-dependent value functions for off-policy evaluation, 2024.